

Final Exam

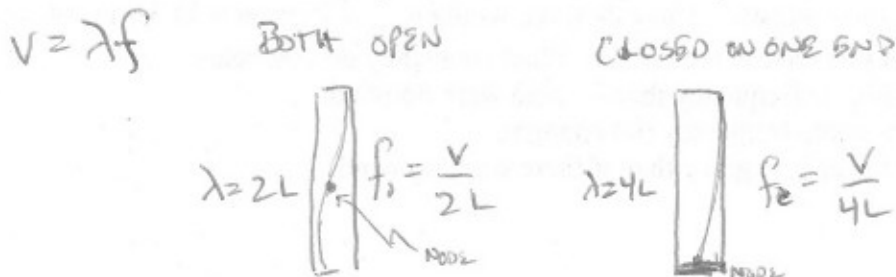
December 18, 2007

This is a closed book examination. You may use a 3x5 card with equations on it. There is extra scratch paper available. Explanations must be included with all answers – even multiple-choice questions where the explanation is worth 75% of the possible points. Show your thoughts!

A general reminder about problem solving:

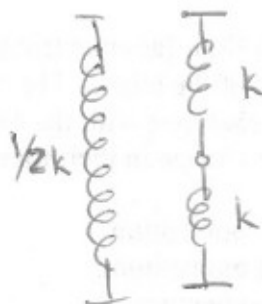
1. Draw a picture then create a simplified free body diagram with all forces
2. Write down what you know including coordinate frame
3. Write down what you don't know and/or want to know
4. List mathematical relationships
5. Simplify and solve
6. Check your answer – Is it reasonable? Are units correct?
 - Show all work! Use extra paper if needed.

- 1) [8 pts] If you blow air over the top of a fairly large drinking straw you can hear a fundamental frequency due to a standing wave being set up in the straw. You want to decrease the fundamental frequency so the fundamental is $\frac{1}{2}$ of the original fundamental frequency. NOTE: A straw is open on both ends.
- a) You should place your finger over the bottom of the straw.
 - b) You should cut the straw in half. ← increases the fundamental frequency
 - c) You should cut the straw in half and then place your finger over the bottom of the straw.
 - d) You should fill the straw with a gas (like He) so the velocity of sound increases by two.



- 2) [8 pts] You measure the spring constant of a purple spring to be k_p . You decide to cut this purple spring in half. The new spring has a spring constant that is

- a) Less, $\frac{1}{4}k_p$.
- b) Less, $\frac{1}{2}k_p$.
- c) The same, k_p .
- d) Greater, $2k_p$.
- e) Greater, $4k_p$.



$$k_{\text{eff}} = \left(\frac{1}{k} + \frac{1}{k} \right)^{-1} = \frac{k}{2}$$

Spring constant is reduced when you put springs in parallel

- 3) [8 pts] You and a friend are on the roof of the science lab. You kick a soccer ball horizontally at the same time that your friend drops a bowling ball. You can ignore air drag but you can not ignore the fact that the bowling ball is much more massive than the soccer ball. Which ball hits the ground first?
- The soccer ball hits the ground first.
 - The bowling ball hits the ground first.
 - Both balls hit at the same time.
 - Not enough information given.

$\downarrow g$ is same for both objects
(no other forces on the objects)

- 4) [8 pts] You have a ring and a solid disk that both have the same radius and are free to rotate (spin) about their axis. The disk has twice the mass of the ring. If you apply the same torque to these objects for the same angle, which object is spinning faster?

- The ring
- The disk
- They both spin at the same rate

$$I_D = \frac{1}{2} m_D r^2 \quad I_R = m_R r^2$$

$$I_D = I_R \text{ since } m_D = 2m_R$$

$$\Delta E = \int \vec{\tau} \cdot d\vec{\theta}$$

same change in energy

$$E = \frac{1}{2} I \omega^2 \text{ so same } \omega$$

- 5) [8 pts] You are outside on a very windy day watching a soccer game when the referee blows their whistle. The gale force wind ($v_{\text{wind}} = 75 \text{ mph} = 33.3 \frac{\text{m}}{\text{s}}$) is blowing in your face as you look at the referee. What frequency do you hear?

- A higher frequency than if there were no wind.
- The same frequency (no change).
- A lower frequency than if there were no wind.

Both observer and source are stationary so there is no doppler shift - and hence no change in f .
(There is a change in the λ and speed of the sound)
 $v = \lambda f$

- 6) [8 pts] You push two boxes across a smooth floor (assume frictionless). The two boxes are identical except one has twice the mass of the other. The heavier box is moving twice as fast as the lighter box. You push each box with the same force for the same amount of time. Which box has the greater change in their momentum after you are finished pushing?

- The heavy box has a greater change in momentum.
- The lighter box has a greater change in momentum.
- Both boxes have the same change in momentum.

$$\int \vec{F} \cdot dt = \Delta \vec{p} \quad \text{Impulse} = \Delta \text{momentum}$$

SAME change in momentum for both boxes.

- 7) [8 pts] You push a large merry-go-round so that it is traveling $\bar{\omega} = 10 \hat{k} \frac{\text{rad}}{\text{sec}}$. After you stop pushing, the merry-go-round comes to a stop in almost 16 revolutions ($\Delta\theta = 100 \text{ rad}$). You know that the moment of inertia of this merry-go-round is 1000 kg m^2 . If air drag is negligible what is the frictional torque in the spinning mechanism?

- a) $\bar{\tau} = -1000 \hat{k} \frac{\text{kg m}^2}{\text{s}^2}$
 b) $\bar{\tau} = -500 \hat{k} \frac{\text{kg m}^2}{\text{s}^2}$
 c) $\bar{\tau} = -100 \hat{k} \frac{\text{kg m}^2}{\text{s}^2}$
 d) $\bar{\tau} = 100 \hat{k} \frac{\text{kg m}^2}{\text{s}^2}$
 e) $\bar{\tau} = 500 \hat{k} \frac{\text{kg m}^2}{\text{s}^2}$
 f) $\bar{\tau} = 1000 \hat{k} \frac{\text{kg m}^2}{\text{s}^2}$

Use $\sum \bar{\tau} = I \bar{\alpha}$ or $\Delta E = \int \bar{\tau} d\theta$

$\omega = \omega_0 + \alpha t$
 $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
 solve to find $\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$
 $\omega = 0 \Rightarrow \dots$
 $\bar{\tau} = I \alpha = \left(\frac{-\omega_0^2}{2\Delta\theta} \right) I$

$E_f - E_i = \bar{\tau} \Delta\theta$ * Has to be negative (opposite to $\bar{\omega}$ so it slows)
 $0 - \frac{1}{2} I \omega_0^2 = \bar{\tau} \Delta\theta$
 $\frac{-I \omega_0^2}{2\Delta\theta} = \bar{\tau}$
 $\bar{\tau} = - \frac{1000 \text{ kg m}^2 \cdot (10/\text{s})^2}{2(100)} = -500 \text{ N m}$

- 8) [8 pts] You push two boxes across a carpeted floor (include friction). The two boxes are identical except one has twice the mass of the other. You release each box when they are both traveling the same speed. Which box has a smaller stopping distance?

- a) The heavy box has a shorter stopping distance.
 b) Both boxes have the same stopping distance.
 c) The lighter box has a shorter stopping distance.

X: $-F_f = ma$

Y: $F_N - F_g = 0$

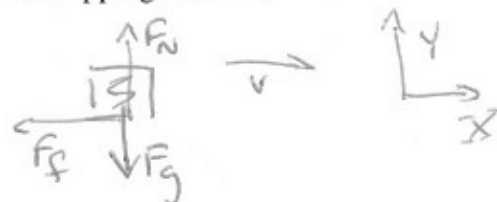
$F_N = F_g = mg$

$F_f = \mu_k F_N = \mu_k mg$

$-\mu_k mg = ma$

$-\mu_k g = a$

independent of mass — so both have same acceleration



Choose 2 out of the next 3 problems to complete. I will only grade the 2 problems indicated.

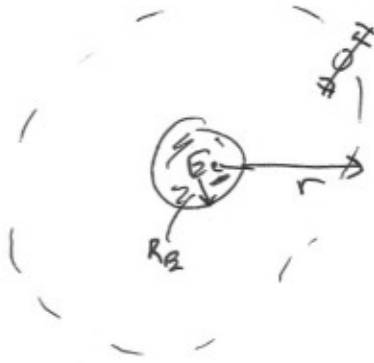
- 9) [12 pts] You want to place a 700kg satellite into a geosynchronous orbit around the Earth. You know that the $R_E = 6.37 \times 10^6 \text{ m}$, $M_E = 5.97 \times 10^{24} \text{ kg}$ and $G = 6.673 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$. Assume a circular orbit and ignore air drag (this is an idealized problem).

- a) What is the radius of a geosynchronous orbit? NOTE: The orbital period is 24 hours = 86.4 ks.
 b) How fast would you need to launch the satellite to reach this radius? NOTE: The escape velocity for an object on the surface of the Earth is roughly 11.2 km/s.
 c) How much energy would be needed to move this satellite to a geosynchronous orbital radius? BONUS: How much more energy is needed to be placed in a stable orbit?

- 10) [12 pts] A mass of 100g is hanging from a spring. When you add another 100g the spring stretches 5cm so it is now 25cm long.

- a) How much mass should you hang from the spring so that it oscillates vertically at $f = 1 \text{ Hz}$?
 b) If instead of oscillating vertically you start the "pendulum" (with added mass from part a) swinging what is this oscillation frequency?

9)



$$T = 24 \text{ hours} = 86.4 \times 10^3 \text{ sec}$$

$$\Sigma F = ma_c$$

$$\frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$\frac{GM_E}{r^2} = \frac{(2\pi)^2 r^3}{T^2} \quad \text{so} \quad \frac{GM_E}{(2\pi)^2} = \frac{r^3}{T^2}$$

$$r_0 = \left(\frac{T^2 GM_E}{(2\pi)^2} \right)^{1/3}$$

(a)

$$r_0 = 42 \times 10^6 \text{ m} \quad (6 \times R_E \text{ far away when compared to } 400 \times 10^3 = 4 \times 10^5 \text{ m for space station orbit})$$

(b)

$$E_i = E_f$$

$$P_{E_i} = \frac{-GM_E m}{r_E}$$

$$\frac{1}{2} m v_i^2 - \frac{GM_E m}{R_E} = 0 - \frac{GM_E m}{r_0}$$

$$\text{so} \quad v_i = \left[2GM_E \left(\frac{1}{R_E} - \frac{1}{r_0} \right) \right]^{1/2}$$

$$v_i = 10.3 \text{ km/s}$$

(less than v_{esc} so this is reasonable)

(c) Energy needed is $\frac{1}{2} m v^2$ (ΔKE)

$$= 3.7 \times 10^{10} \text{ J}$$

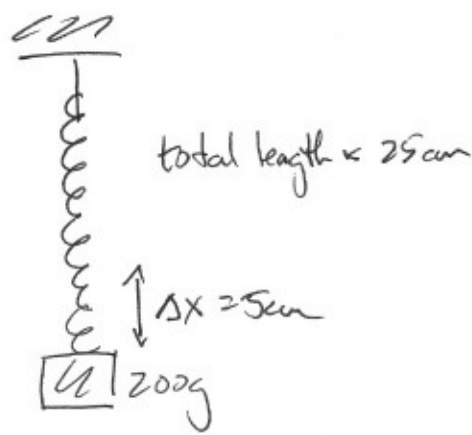
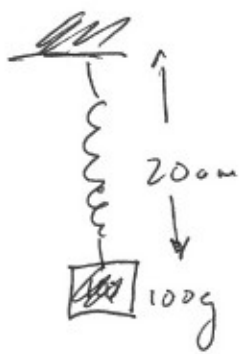
Bonus

$$\text{Add orbital } KE \quad \frac{1}{2} m v_0^2 \quad v_0 = \frac{2\pi r_0}{T} = 3,072 \text{ m/s}$$

$$= 9.44 \times 10^6 \text{ J}$$

↑ small potatoes compared to energy needed to boost into orbit!

10)



$$F = k \Delta x$$

$$k = \frac{F}{\Delta x} = \frac{(0.1 \text{ kg})(9.81 \text{ m/s}^2)}{0.05 \text{ m}}$$

$$k = 19.62 \text{ kg/s}^2$$

(a) Springing $\omega = \sqrt{\frac{k}{m}}$ $\omega = 2\pi f$

$$m = \frac{k}{\omega^2} = \frac{k}{(2\pi)^2 f^2}$$

$m \approx 500 \text{ g}$ total $\frac{g}{s}$.

(b) length = 20cm + Δx

$$\Delta x = \frac{(0.4 \text{ kg})(9.81 \text{ m/s}^2)}{19.62 \text{ kg/s}^2} = 0.2 \text{ m} \quad (20 \text{ cm})$$

Swinging $\omega = \sqrt{\frac{g}{l}}$ $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \left(\frac{g}{l} \right)^{1/2} \quad l = 0.4 \text{ m}$$

$f \approx 0.79 \text{ Hz}$

11) [12 pts] You observe a car as it approaches and then leaves an intersection where it had to obey a stop sign. You watch as the car travels 10m/s (22.5 mph) for 10 seconds, takes 5 seconds to stop, waits 10 seconds for a nurse walking a cat to cross the street and then resumes moving forward, accelerating at 1m/s² for 10 seconds and then continues at this final velocity for another 5 seconds at which time you lose sight of the car. NOTE: Label each axis with correct units.

- Draw the position of the car as a function of time.
- Draw the velocity of the car as a function of time.
- Draw the acceleration of the car as a function of time.

